ME 5643 FINAL PROJECT

DENSITY METER

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ABSTRACT

The density of matter is defined as the ratio of mass to volume. We have constructed an instrument to calculate the density of various solids and liquids which is operated by the Basic Stamp 2 (BS2). The instrument consists of a microcontroller, a force sensor, an ultrasonic sensor, and a servo motor actuator, which are all housed in a unique sample chamber. A one liter glass beaker is mounted directly over a force sensor, which is properly interfaced with the BS2. The ultrasonic sensor is aligned directly over the top of the beaker, which measures the height of liquid in the beaker. The BS2 relates the change in liquid height to the object volume through appropriate calculations. The density of solids is determined by measuring a difference in mass upon immersing it in a given volume of water, and dividing that mass by the volume change (corresponding to the volume of the object) in the measurement chamber. The density of liquids is determined by measuring the force exerted by the liquid (and hence its mass), and dividing that mass by the liquid height (corresponding to its volume) in the measurement chamber. This instrument is intended for educational purposes, and can be implemented in middle school science classrooms to demonstrate and teach the concept of density and how it is determined.

CONTENTS

INTRODUCTION

The concepts of science, technology, engineering, and mathematics are often difficult for children to grasp. With the assistance of devices that permit hands-on learning, educators can enhance the learning process for students and enable them to contextualize the topics at hand. Mechatronics is an excellent tool for just this purpose: children are often so fascinated by the mechanical workings of robotic instruments that the resistance barrier to learning about even the most complex scientific concepts virtually disappears.

One of the more obscure topics that children must be introduced to in their middle school education is matter and its properties. Specifically, one of the more important basic properties in the studies of chemistry, physics, and materials science is density. The density of matter in all its states: liquid, gas, and solid, is determined by the ratio of the material's mass to its volume. Children may be familiar with this topic in relation to whether an object will sink or float in water, or may relate to the physical sensation of holding a bowling ball compared to a Styrofoam ball. Upon directly experimenting with different materials and visualizing how the density is calculated brings a child's understanding to a completely new level.

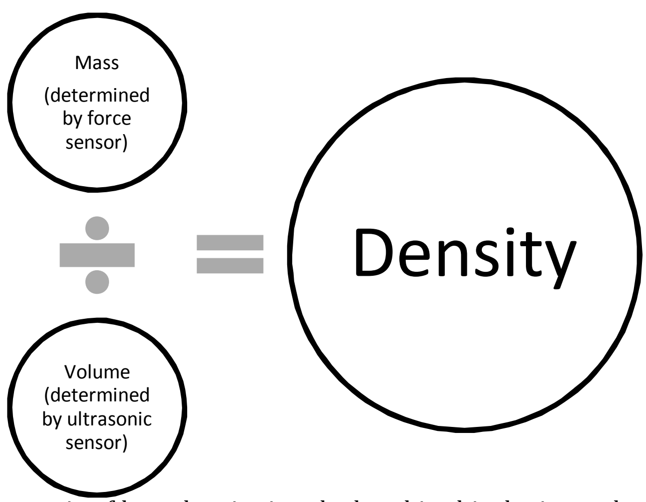

Figure 1: Simple schematic of how density is calculated in this device and what sensors are involved.

The device designed herein aims to provide students with an interactive method for understanding density of liquids and solids. Experiments and measurements performed with this device can be appropriately scaled for applications in classrooms ranging from elementary and middle schools (comparing the density of solids to liquids or evaluating why certain objects sink or float, for example) all the way up to high school chemistry classes (measurement of the density of homogeneous and heterogeneous mixtures, for example). Students will be able to chose materials, mount them on the device, immerse them in water, and observe how the density meter uses appropriate sensors to determine the mass, volume, and density values. The schematic in Figure 1 summarizes the components used and how density is determined in this device.

MATERIALS AND METHODS

SENSORS AND MATERIALS

Two sensors are implemented in this device. The first of these is Parallax's ultrasonic PING)))™ sensor. This sensor provides a reliable measurement of distance of stationary objects. The ultrasonic sensor functions by emitting a 40 kHz tone and measuring the time it takes to receive the echo of this signal, as illustrated in Figure 2. The distance range for this sensor is 2 cm – 3.3 m, which is more than sufficient for the scope of this device. The PING)))™ sensor is interfaced with the BS2 via a 3-pin connection.

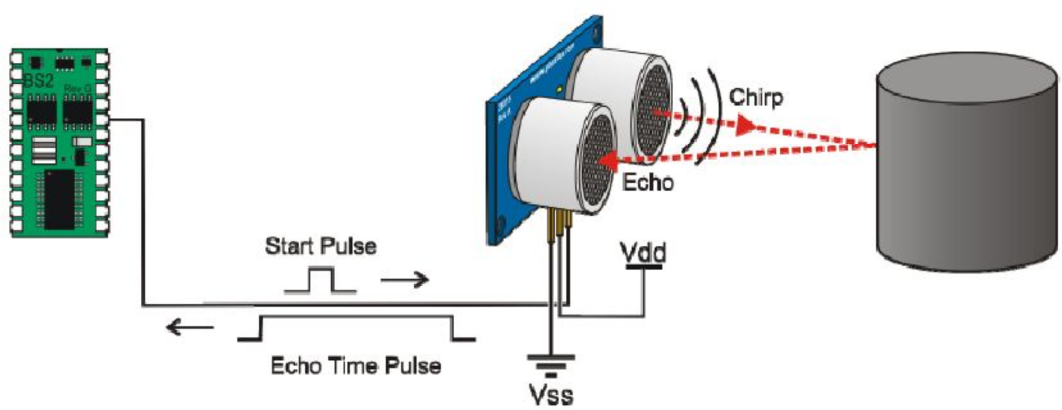

Figure 2: Functional mechanism of the ultrasonic PING)))™ sensor.

The second sensor that is used in this device is a Tekscan FlexiForce® force sensor. This thin and flexible sensor (depicted in Figure 3) is capable of measuring force exerted by the weight of objects 0-25 lbs (model A201-25)[i]. The sensitive area of the FlexiForce®

Figure 3: FlexiForce® sensor used to measure mass acts as a variable resistor.

sensor is composed of two adjacent faces covered in a conductive polymer. Upon application of force, these faces come in contact with one another, resulting in an increase in conductivity (decreasing resistivity). This sensor therefore acts as a type of variable resistor, with a maximum resistance (> 5 MΩ) in the absence of a detectable load and a mimimum resistance (5 KΩ) upon maximum load of 25 lbs. The FlexiForce® is interfaced with the BS2 via a 3-pin connection.

A continuous servo motor is used as an actuator in the system. Servo motors are DC motors with feedback position control. This actuator has a torque value of 3.4 kg·cm, which is sufficient for the size of the objects that we expect to measure with this density meter. The servo motor is interfaced with the BS2 via a 3-pin connection.

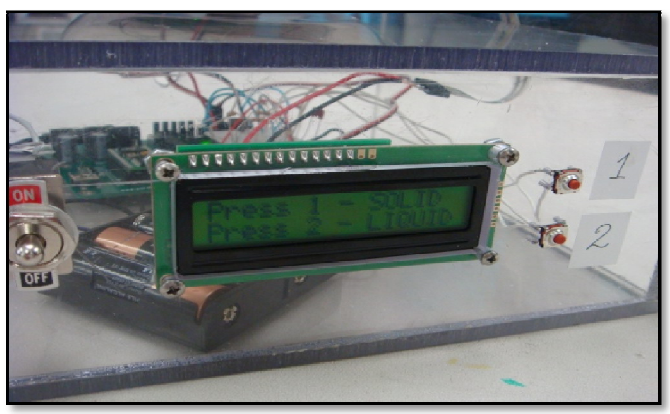

Figure 4: Liquid crystal display on the front panel of the density meter.

A liquid crystal display (LCD) component is used as a user interface to display the measurement results obtained, as seen in Figure 4. The LCD is a Parallax 2x16 serial LCD, and is interfaced with the BS2 via a 3-pin connection.

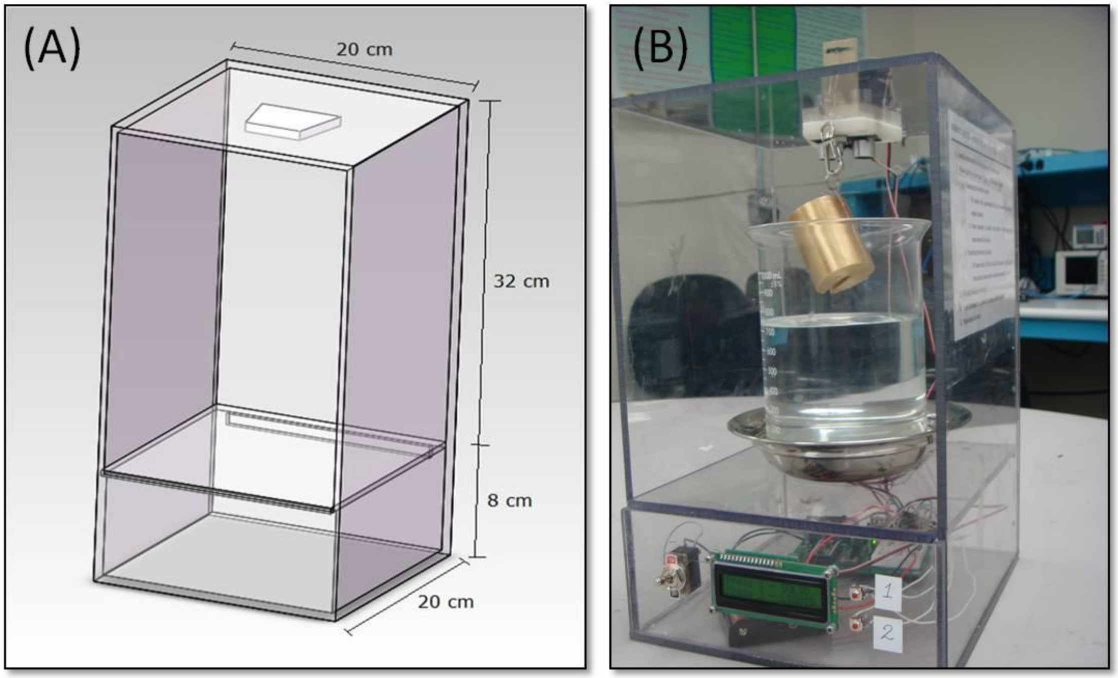

Figure 5: Density meter structure, composed from Plexiglas. (A) SolidWorks diagram of the Plexiglas housing for the density meter, complete with dimensions. (B) Photo of the density meter, including the structure and the measurement chamber containing a 1 l glass beaker.

The structure which houses the density meter is composed of Plexiglas panels which have been assembled with glue. Its dimensions are outlined in Figure 5A. Holes were drilled in several locations in order to allow for appropriate wiring between sensors and the BS2, which is housed in the bottom compartment of the structure.

BILL OF MATERIAL

Table 1: Bill of material for prototype construction, including the estimated cost per unit and the total cost for each material. The overall prototype cost is estimated to be $238.82.

	Material	**Cost per unit ($)**	**Quantity**	**Total cost ($)**
1	PING)))™ sensor	$20.00	1	$20.00
2	FlexiForce® sensor	$20.00	1	$20.00
3	5 mm thick Plexiglas	$105.00/m^2	0.376 m^2	$39.63
4	LCD display	$25.00	1	$25.00
5	Servo motor	$13.00	1	$13.00
6	BS2 microcontroller	$100.00	1	$100.00
7	Switch	$3.19	1	$3.19
8	AA battery	$0.50	4	$2.00
9	1 l glass beaker	$15.00	1	$15.00
10	Misc. electrical components	$1.00	-	$1.00
			TOTAL PROTOTYPE COST =	$238.82

Table 1 presents a complete bill of materials used in the construction of the density meter. The estimated sum of these materials is approximately $240 per unit. It is expected that this per unit cost would decrease significantly if this device were to be mass produced. The reduction is estimated to be approximately 50 %, making the mass production cost per unit around $120.

MATHEMATICAL BACKGROUND AND CALIBRATION

The data obtained from the ultrasonic and force sensors required some mathematical manipulation in order to provide the user with comprehensive, useful information. As described previously, the ultrasonic sensor determines the time it takes for an emitted tone to travel to an object and return to the sensor. This sensor therefore gives an output of time if no further calculations are applied. In order to convert the travel time to distance, the following calculations were performed. Distance is related to time, and the speed of sound in air (at 22 °C), by Equation 1:

Equation 1 $$D = \frac{c\, x\, t}{2}$$

where d is distance in cm, c is the speed of sound in air (34480 cm/s), and t is the travel time in seconds.

In order to manipulate the data from the FlexiForce® sensor a calibration curve was necessary. This was done by measuring the *RCTime* value given by the FlexiForce® sensor in a parallel RC circuit with a 0.01 μF capacitor, as is described further in the following section. *RCTime* values were obtained for precisely measured masses of water, in a range of approximately 100 to 1500 g. Resistance was determined from measured *RCTime* values, according to Equation 2:

Equation 2 $$RCTime = 635R\ x\ C$$

where *RCTime* is given in units of 635 ms, C is the capacitance in units of μF, and R is the resistance in kΩ. R is obtained by simply rearranging Equation 2. Plotting R versus mass gives an exponentially decreasing curve, as shown in Figure 6.

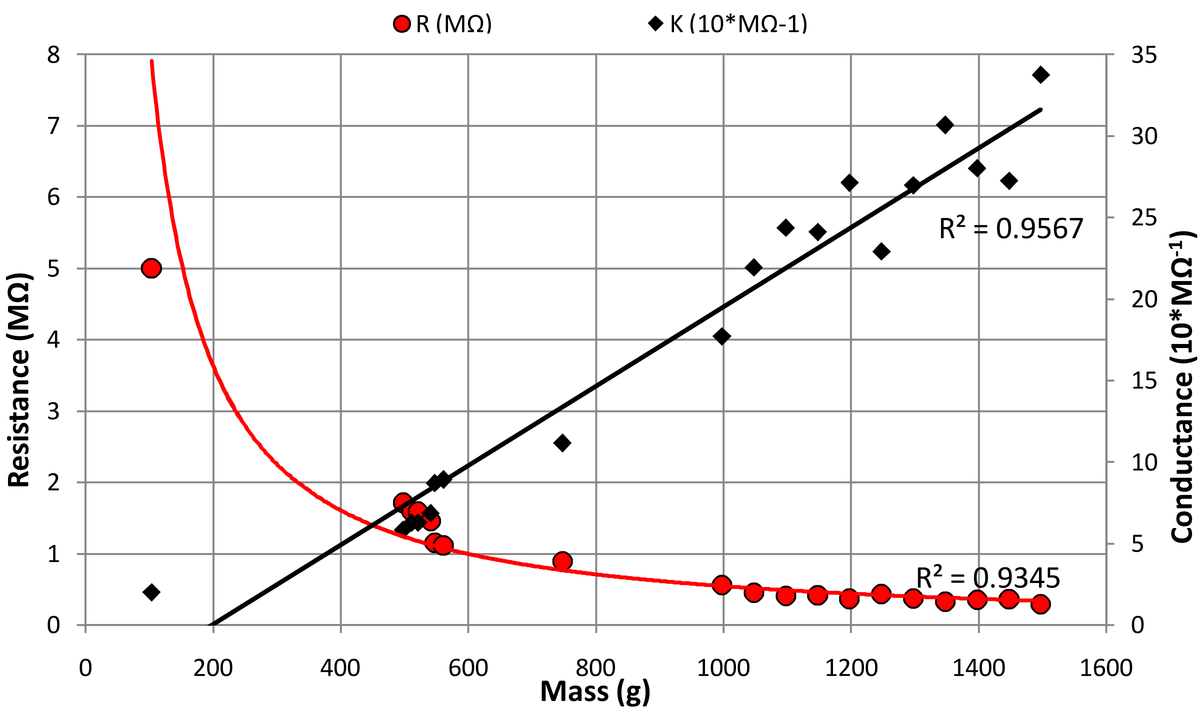

Figure 6: Calibration curve for the FlexiForce® sensor. Resistance vs. mass is shown in red, while the linearization of this data (conductance vs. mass) is shown in black. An exponential regression was applied to the resistance curve and was shown to have a fit of 0.9345. A linear regression was applied to the conductance curve and a fit of 0.9567 was obtained.

The linearization of resistance was obtained by taking the inverse of this value (conductance, or *K*), as per Equation 3, and plotting it against the mass.

Equation 3 $$K = \frac{1}{R}$$

The linear fit shown in Figure 6 is used to obtain the mass value experimentally. The linear regression of the calibration data had the following relationship:

Equation 4 $$M = 41.15K + 196.86$$

where M represents the mass in grams and K is the conductance in $10^{*}M\Omega^{-1}$. The relation of the raw data output, *RCTime*, to the mass is therefore given by Equation 5:

Equation 4 $$M = 41.15K + 196.86$$

$$= 41.15\left(\frac{1}{R}\right) + 196.86$$

Equation 5 $$= 41.15\left(\frac{635C}{RCTime}\right) + 196.86$$

where C = 0.01 μF for the selected capacitor. Mass was determined experimentally for 20 samples of known mass, and percent error was determined according to Equation 6. The average percent error was determined to be 7.33 %. All relevant calibration data is presented in Table 2.

Equation 6 $$\%\ error = 100\ \left(\frac{experimental\ mass - actual\ mass}{actual\ mass}\right)$$

Table 2: Calibration data for 20 samples of known mass. Percent error is calculated by comparing experimentally determined mass to the actual mass, and has an average value of 7.33 %.

	Actual mass (g)	**RCTIME**	**R (MΩ)**	**K (10*MΩ^{-1})**	**Experimental mass (g)**	***% error***
1	103.2	65535	5.00	2.00	279.2	-
2	497.6	10887	1.71	5.83	436.9	*12.2*
3	511.3	10130	1.60	6.27	454.8	*11.0*
4	521.3	10105	1.59	6.28	455.5	*12.6*
5	540.9	9270	1.46	6.85	478.8	*11.5*
6	547.6	7306	1.15	8.69	554.5	*1.3*
7	560.7	7117	1.12	8.92	564.1	*0.6*
8	747.6	5689	0.90	11.16	656.2	*12.2*
9	997.6	3589	0.57	17.70	925.1	*7.3*
10	1047.6	2896	0.46	21.93	1099.2	*4.9*
11	1097.6	2608	0.41	24.35	1198.8	*9.2*
12	1147.6	2636	0.42	24.09	1188.4	*3.6*
13	1197.6	2341	0.37	27.13	1313.1	*9.6*
14	1247.6	2773	0.44	22.90	1139.2	*8.7*
15	1297.6	2355	0.37	26.96	1306.5	*0.7*
16	1347.6	2071	0.33	30.66	1458.6	*8.2*
17	1397.6	2268	0.36	28.00	1349.3	*3.5*
18	1447.6	2332	0.37	27.23	1317.4	*9.0*
20	1497.6	1882	0.30	33.74	1585.4	*5.9*

BASIC STAMP 2 CIRCUITRY

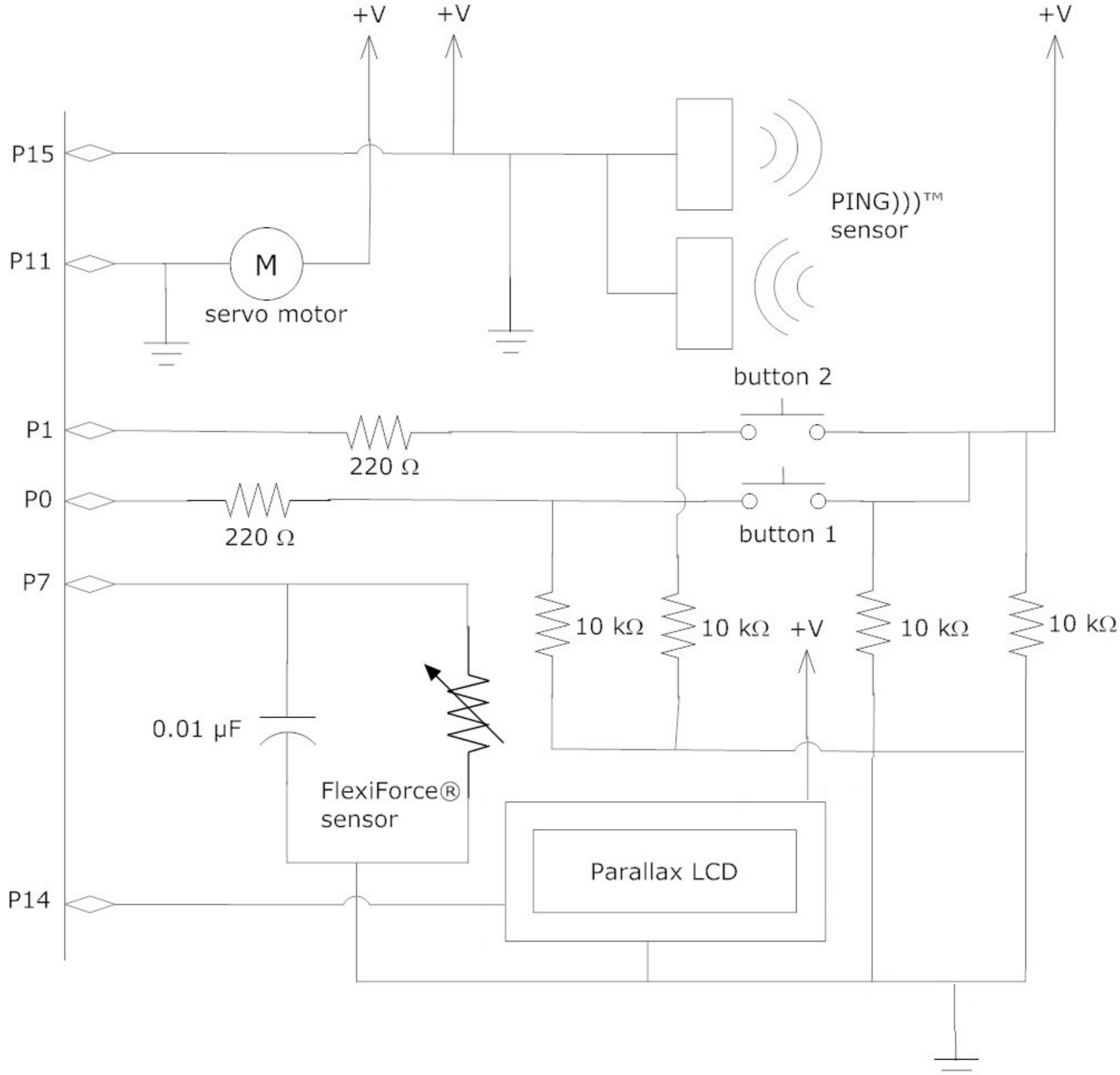

Figure 7: Complete circuitry for the density meter, where +V corresponds to 6 V.

PBASIC PROGRAM

```
' {$STAMP BS2}
' {$PBASIC 2.5}

' -----[ Declaration of variables ]----------------------------------------

'Mass, height, volume, and density
m1          VAR   Word
m2          VAR   Word
h1          VAR   Word
h2          VAR   Word
vol         VAR   Word
density     VAR   Word

'Ultrasonic Sensor Variables
USpin       CON   15
CmConstant  CON   22597
mmDistance  VAR   Word
time        VAR   Word

'Flexiforce Sensor Variables
rawForce    VAR   Word
FFpin       CON   7

'Conditions
material    VAR   Bit
dropped     VAR   Bit
filled      VAR   Bit

'Variables used for calculation purposes
K           VAR   Word
K1          VAR   Word
K2          VAR   Word


' -----[ Main Menu ]-------------------------------------------------------
main:

dropped = 0                                               'Solid not loaded
filled = 0                                                'Liquid not filled

PAUSE 1000

DO

SEROUT 14, 84, [128,"Press 1 - SOLID "]                   'LCD instruction
SEROUT 14, 84, [148,"Press 2 - LIQUID"]

IF IN0 = 1 THEN                                        'If button 1 is pressed, then
material = 0                                           'Material defined to be solid
GOTO height                                               'Measure initial height h1

ELSEIF IN1 = 1 THEN                                    'If button 2 is pressed, then
material = 1                                       'Material is defined to be liquid
GOTO weight                                               'Measures initial mass m1
```

```
ENDIF

LOOP

' -----[ Height Measurements ]-----------------------------------------------
height:

IF material = 0 THEN                                'If the material is solid, then
PULSOUT USpin, 5                                         'measure the initial height
PULSIN USpin, 1, time
h1 = CmConstant ** time                                  'height in mm

SEROUT 14, 84, [22,12]                                   'Clear LCD screen
SEROUT 14, 84, [128,"Mount specimen"]
SEROUT 14, 84, [148,"then press 1"]
PAUSE 250
 DO
 IF IN0 = 1 THEN              'Press button 1 to measure the initial mass m1
 GOTO weight
 ENDIF
 LOOP

ELSEIF material = 1 THEN                      'Else if the material is liquid, then
PULSOUT USpin, 5                                      'measure the initial height h1
PULSIN USpin, 1, time
h1 = CmConstant ** time

SEROUT 14, 84, [22,12]     'After m1 is measured, fill the beaker with liquid
SEROUT 14, 84, ["Fill the beaker then press 1"]

DO
IF IN0 = 1 THEN                       'Press button 1 after beaker is filled
filled = 1
GOTO weight                                    'Go measure the final mass m2
ENDIF
LOOP

ENDIF



' -----[ Running the Motor ]-------------------------------------------------
motor:

SEROUT 14, 84, [22,12]
 SEROUT 14, 84, [128,"Press 2 to drop"]            'Slowly lowering the solid
                                                      to the bottom of beaker
SEROUT 14, 84, [148,"then press 1"]
PAUSE 250

DO

 DO
 IF IN1 = 1 THEN                   'Press button 2 to lower the solid slowly
  FOR K = 1 TO 15
  PULSOUT 11, 850
```

```
  PAUSE 20
  NEXT
 ENDIF
 IF IN0 = 1 THEN             'Press button 1 when solid reached the bottom
 dropped = 1
 GOTO weight                           'Proceed to measure the final mass m2
 ENDIF
 LOOP UNTIL (IN1 = 0)

LOOP

' -----[ Mass Measurements ]-------------------------------------------------
weight:

IF material = 0 THEN                                  'For solid
HIGH FFpin                                         'Measure mass from Flexiforce
PAUSE 2
RCTIME FFpin, 1, rawForce
K = 63500/rawForce                                  'Conductance 1/R
K1 = rawForce/100
K2 = (635//K1) * 41/K1                      'K2 = fraction portion of (41 * K)
m2 = 41*K + 197 + K2          'mass is determined from conductance (linear)

IF dropped = 0 THEN                          'If solid isn't dropped yet, then
m1 = m2                                    'this measurement is initial mass m1
GOTO motor                             'Use the motor to slowly lower the solid
ELSEIF dropped = 1 THEN                     'Else if the solid is lowered, then
GOTO volume                                  'this measurement is final mass m2
ENDIF

ELSEIF material = 1 THEN                              'For liquid
HIGH FFpin                                         'Measure mass from Flexiforce
PAUSE 2
RCTIME FFpin, 1, rawForce
K = 63500/rawForce
K1 = rawForce/100
K2 = (635//K1) * 41/K1                                'If beaker is filled, then
m2 = 41*K + 197 + K2                         'this measurement is final mass m2

IF filled = 0 THEN                        'If the beaker isn't filled yet, then
m1 = m2                                    'this measurement is initial mass m1
GOTO height                                   'Go measure the initial height h1
ENDIF

GOTO volume

ENDIF


' -----[ Volume Calculations ]-----------------------------------------------
volume:

PULSOUT USpin, 5       'Run the ultrasonic sensor to measure final height h2
PULSIN USpin, 1, time

IF material = 0 THEN                                  'Volume for solid
h2 = CmConstant ** time           'height in mm, and is converted to cm below
```

```
vol = ((25*3*(h1-h2)) + ((h1-h2)*7/2))/10          'volume = (r^2)(3)(h1-h2) +
(r^2)(0.14)(h1-h2)

ELSEIF material = 1  THEN                          'Volume for liquid
h2 = CmConstant ** time
vol = ((25*3*(h1-h2)) + ((h1-h2)*7/2))/10

ENDIF

GOTO results

' -----[ Display Results ]-------------------------------------------------
results:

density = (m2 - m1)/vol                            'Calculation for density
K = (m2 - m1) // vol                               'Remainder of density
K1 = K*10 / vol                                       'K1 = first decimal digit
of density value
K = K*10 // vol
K2 = K*10/ vol                                       'K2 = second decimal digit
of density value

PAUSE 250
SEROUT 14, 84, [22,12]                             'Clear LCD

DO
SEROUT 14, 84, [128, "To see result"]
SEROUT 14, 84, [148, "Press 1"]
LOOP UNTIL (IN0 = 1)

PAUSE 250
SEROUT 14, 84, [22,12]

DO
SEROUT 14, 84, [128, "Mass: ", DEC5 m2-m1, "g"]
SEROUT 14, 84, [148, "Press 2 for more"]
LOOP UNTIL (IN1 = 1)

PAUSE 250
SEROUT 14, 84, [22,12]

DO
SEROUT 14, 84, [128, "Volume: ", DEC5 vol, "ml"]
SEROUT 14, 84, [148, "Press 2 for more"]
LOOP UNTIL (IN1 = 1)

PAUSE 250
SEROUT 14, 84, [22,12]

IF material = 0 THEN
SEROUT 14, 84, [128, "Density:", DEC2 density, ".", DEC1 K1, "g/ml"]
SEROUT 14, 84, [148, "Press 2: remove "]

DO
LOOP UNTIL (IN1 = 1)
SEROUT 14, 84, [22,12]
SEROUT 14, 84, [128, "Press 1: lift"]
```

```
SEROUT 14, 84, [148, "Press 2: rerun "]
PAUSE 250

DO
 DO
  IF IN0=1 THEN                     'Press button 1 to slowly lift the solid
  FOR K = 1 TO 20
  PULSOUT 11, 650
  PAUSE 20
  NEXT
  ENDIF
 LOOP UNTIL (IN0 = 0)
 IF IN1 = 1 THEN                    'Go back to main when button 2 is pressed
 GOTO main
 ENDIF
LOOP

ELSEIF material = 1 THEN
SEROUT 14, 84, [128, "Density:", DEC2 density, ".", DEC1 K1, "g/ml"]
SEROUT 14, 84, [148, "Press 1: rerun  "]

DO                                  'Go back to main when button 1 is pressed
IF IN0 = 1  THEN
GOTO main
ENDIF
LOOP

ENDIF
```

RESULTS AND DISCUSSION

The PBASIC code works as designed. Since the final density result is a factor of the measurement of several sensors and various calculations, it is subject to error. As seen by the calibration data presented in Table 2, the FlexiForce® sensor contributes to approximately 7 % error. The error of the ultrasonic sensor is estimated to be approximately 5 %.

Table 3: Actual versus experimental mass, volume, and density for water and a 500 g metal alloy standard weight.

Material	Actual			Experimental			% error
	Mass (g)	Volume (ml)	Density (g/ml)	Mass (g)	Volume (ml)	Density (g/ml)	
Water	750	750	1.0	582	706	0.8	*17.6*
500 g weight	500	110	4.5	482	109	4.4	*2.7*
				Average % error =			*10.1*

The density meter has been used to evaluate the density of water and a 500 g standard metal alloy weight to evaluate the overall accuracy of the instrument. The actual and experimental mass, volume, and density for these materials are presented in Table 3. The percent error in experimental and actual density values for the standards measured was determined by Equation 6. The average percent error of the density meter is approximately 10 %, which is a result of the compounded error of the force sensor, the ultrasonic sensor, and the mathematical approximations made in the PBASIC software program.

This device produces the most accurate results when measuring samples that fall within certain mass and volume limitations. The ideal mass range is from 200 – 1400 g. The sample volume is restricted by the size of the measurement beaker, which has a radius of 5 cm and a total volume of 1 l.

CONCLUSION

The overall accuracy of the density meter is acceptable if this instrument is to be used for educational purposes. For the density meter to be used as a reliable measurement device, however, the accuracy of the mass and volume determination must be enhanced. For improved accuracy, further mathematical manipulations are required. Much of the inconsistency in the calculations generated by this density meter is due to the limitations of the PBASIC software in handling decimal values and the size of the variables it can store. A more accurate determination of the density may be able to be achieved by using a software has higher numerical processing capabilities.

The structure and interface of the device enable easy use and straightforward determination of data. In addition, the transparency of the Plexiglas material from which the density meter is constructed allows students to see the Basic Stamp and all circuitry, providing them insight as to how such a device is assembled. This density meter is therefore an excellent learning platform, one that can even be used to construct a more comprehensive measurement device, with the potential to add a digital thermometer, a pH meter, or other sensors relevant to the study of chemistry.

The instruction of scientific concepts and material properties is much more effective when students are able to perform experiments that contextualize these lessons. The density meter that has been designed and built is an excellent tool to teach students about the measurement of mass, volume, and of course calculation of density of both liquids and solids.

REFERENCES

[i] Tekscan FlexiForce® Sensor User Manual. Tekscan, Inc. South Boston, MA. 2009.