

# Pendulum Periods

## I. Introduction

In this experiment, students will become familiar with the mechanics of a pendulum and the physics behind it. A pendulum is a mass that is attached to a (effectively mass-less) string and then attached to some fixed point, allowing the mass to hang down and have a free range of motion either in two or three dimensions. The mass has the ability to oscillate back and forth.

## II. Background

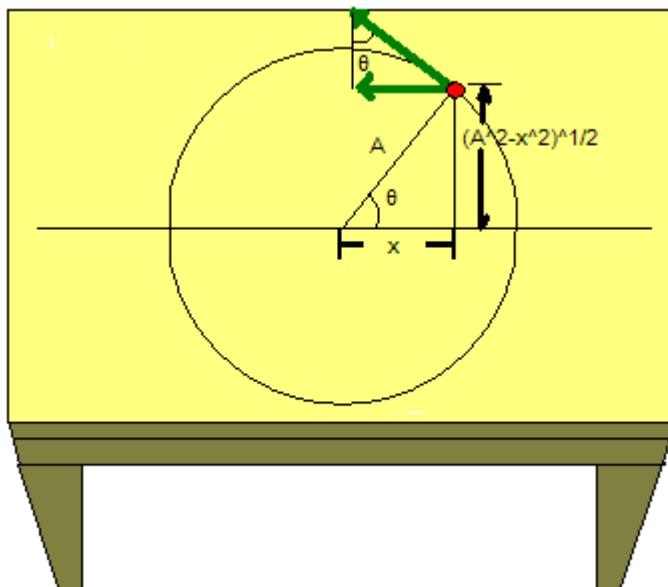
A pendulum has the characteristic of swinging back to nearly the exact height that it started from. Galileo was the first to observe the unique oscillation of the pendulum. Today, this demonstrates conservation of energy, a principle that had not yet been discovered in Galileo's time.

The period of the pendulum is defined as the time it starts swinging from one point to another and back again. Imagine a pendulum moving along the table in a circular path. The velocity will be equal to the circumference of the circle (the distance traveled divided by the time it took) which in this case is the period,  $v = \frac{2\pi A}{T}$  (Note: the radius is A, amplitude). Therefore,

the period is equal to  $T = \frac{2\pi A}{v}$ . The energy stored in motion is:

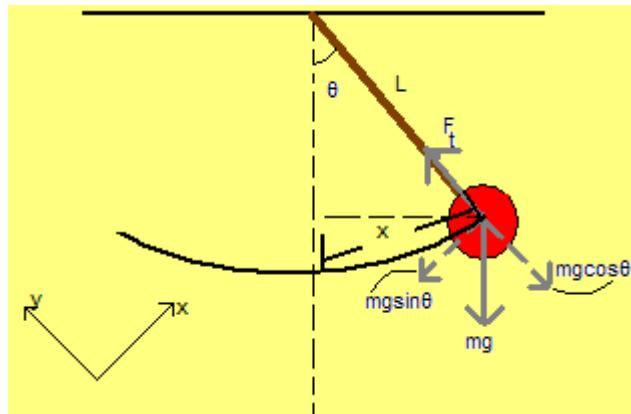
$$E = \frac{1}{2} kA^2 \Rightarrow \frac{1}{2} mv^2 + \frac{1}{2} kx^2 = \frac{1}{2} kA^2 \text{ getting the proportionality of } A \text{ and } v \text{ which will equal}$$

$$\frac{A}{v} = \sqrt{\frac{m}{k}} \text{ combining this formula with the above formula } T = 2\pi \sqrt{\frac{m}{k}}$$



<sup>1</sup> Figure 1 (Top view)

<sup>1</sup> Giancoli, Douglas C. *Physics: Principles with applications* / 5<sup>th</sup> Ed., 1998



^2 Figure 2 (Side view)

Now we need to relate the period to the pendulum based system.  $F = -mg \sin \theta$  (the minus sign indicates the arrow pointing to the negative side, figure 2). For small angles  $F = -mg \sin \theta \approx -mg\theta$  and the arc length is equal to  $x = L\theta$  combining the two formulas  $F \approx -mg \frac{x}{L}$ . According to Hook's Law  $F = -kx$  where  $k = \frac{mg}{L}$ . Going back to the previous equation  $T = 2\pi\sqrt{\frac{m}{k}}$  substitute for  $k$  and we get  $T = 2\pi\sqrt{\frac{L}{g}}$ .

### III. Objective

- Measure the period of a pendulum as a function of amplitude
- Measure the period of a pendulum as a function of length
- Measure the period of a pendulum as a function of bob mass

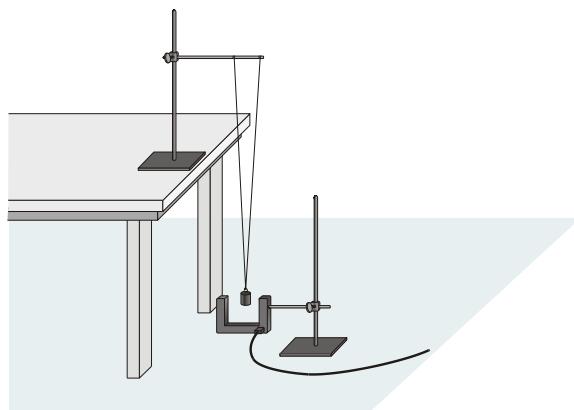


Figure 3

<sup>2</sup> Giancoli, Douglas C. *Physics: Principles with applications* / 5<sup>th</sup> Ed., 1998

## IV. Equipment List

Power Macintosh or Windows PC	String
LabPro or Universal Lab Interface	Two ring stands and pendulum clamp
Logger Pro	Mass Bob
Vernier Photogate	Meter stick
Protractor	Graphical Analysis or graph paper

## V. Experimental Procedure

1. Use the ring stand to hang the any mass from the two strings. Attach the strings to a horizontal rod about 10cm apart, as shown in Figure 3. This arrangement will let the mass swing only along a line, and will prevent it from striking the Photogate. The length of the pendulum is the distance from the point on the rod halfway between the strings to the center of the mass. The pendulum length should be at least 1m.
2. Attach the Photogate to the second ring stand. Position it so that the mass blocks the Photogate while hanging straight down. Connect the Photogate to DG 1 on the Universal Lab Interface or DIG/SONIC 1 on the LabPro.
3. Temporarily move the mass out of the center of the Photogate. Notice the reading in the status bar of Logger Pro at the bottom of the screen, which shows when the Photogate is blocked. Block the Photogate with your hand; note that the Photogate is shown as **blocked**. Remove your hand, and the display should change to **unblocked**. Click  and move your hand through the Photogate repeatedly. After the first blocking, Logger Pro reports the time interval between every other block as the period. Verify that this is so.
5. Now you can perform a trial measurement of the period of your pendulum. Pull the mass to the side about  $10^{\circ}$  from vertical and release. Click  and measure the period for five complete swings. Click . Click the Statistics button, , to calculate the average period. You will use this technique to measure the period under a variety of conditions.

### Part I Amplitude

6. Determine how the period depends on amplitude. Measure the period for five different amplitudes. Use a range of amplitudes, from just barely enough to unblock the Photogate, to about  $30^{\circ}$ . Each time, measure the amplitude using the protractor so that the mass with the string is released at a known angle. Repeat Step 5 for each different amplitude. Record the data in your data table.

### Part II Length

7. Use the method you learned above to investigate the effect of changing pendulum length on the period. Use the any mass given and a consistent amplitude of  $20^{\circ}$  for each trial. Vary the pendulum length in steps of 10 cm, from 1.0 m to 0.50 m. If you have room, continue to a longer length (up to 2 m). Repeat Step 5 for each length. Record the data in the second data table below. Measure the pendulum length from the rod to the middle of the mass.

**Part III Mass**

8. Use the three masses to determine if the period is affected by changing the mass. Measure the period of the pendulum constructed with each mass, taking care to keep the distance consistent from the ring stand rod to the center of the mass, as well as keeping the amplitude the same. Repeat Step 5 for each mass, using amplitude of about  $20^\circ$ . Record the data in your data table

**VI. Analysis/Questions:**

1. Why is Logger *Pro* set up to report the time between *every other* blocking of the Photogate? Why not the time between every block?
2. Using either Graphical Analysis or graph paper, plot a graph of pendulum period *vs.* amplitude in degrees. Scale each axis from the origin (0,0). Does the period depend on amplitude? Explain.
3. Using either Graphical Analysis or graph paper, plot a graph of pendulum period  $T$  *vs.* length  $L$ . Scale each axis from the origin (0,0). Does the period appear to depend on length?
4. Using either Graphical Analysis or graph paper, plot the pendulum period *vs.* mass. Scale each axis from the origin (0,0). Does the period appear to depend on mass? Do you have enough data to answer conclusively?
5. To examine more carefully how the period  $T$  depends on the pendulum length  $L$ , create the following two additional graphs of the same data:  $T^2$  *vs.*  $L$  and  $T$  *vs.*  $L^2$ . Of the three period-length graphs, which is closest to a direct proportion; that is, which plot is most nearly a straight line that goes through the origin?
6. Using Newton's Laws, we could show that for some pendulums, the period  $T$  is related to the length  $L$  and free-fall acceleration  $g$  by

$$T = 2\pi \sqrt{\frac{L}{g}}$$

Does one of your graphs support this relationship? Explain.

**Data Sheet****Part I Amplitude**

Amplitude ( $^\circ$ )	Average period (s)

**Part II Length**

Length (cm)	Average period (s)

**Part III Mass**

Mass (g)	Average period (s)

**EXTENSIONS**

1. From your graph of  $T^2$  vs. L determine a value for  $g$ .
2. Given what you observed in this experiment, write a set of rules for constructing a pendulum clock that is reliable under a variety of temperatures.
3. Try a larger range of amplitudes than you used in Part I. If you did not see a change in period with amplitude before, you should now. Check a college physics textbook for an expression for the period of a pendulum at large amplitudes and compare to your own data.

## References

[1]	Online: <a href="http://www.physicsclassroom.com/Class/vectors/U3L2a.html">http://www.physicsclassroom.com/Class/vectors/U3L2a.html</a> , web site of physicsclassroom.com containing the picture above.
[2]	Giancoli, Douglas C. Physics: Principles with applications / 5 <sup>th</sup> Ed., 1998
[3]	K. Appel, J. Gastineau, C. Bakken, D. Vernier, Physics with Computers. Vernier Software & Technology, Beaverton, Oregon, 3rd Ed., (2003).