

## Simple Harmonic Motion

### Introduction

Simple harmonic motion refers to a body oscillating periodically about an equilibrium position. Familiar examples of such oscillations are a block attached to a spring, the swinging of a child on a playground swing, the motion of a pendulum, and the loudspeaker in a radio.

If a body is experiencing simple harmonic motion, its displacement from its equilibrium position can be described by the relationship

$$y(t) = A \cos(\omega t + \phi)$$

where  $A$ ,  $\omega$ , and  $\phi$  are constants and  $y(t)$  is the displacement of the mass. To give physical significance to these constants, a plot of relationship above is shown in Fig. 1. The amplitude  $A$  of the motion is the maximum displacement in either the positive or negative  $y$  direction. The constant  $\omega$ , called the angular frequency, is determined by  $\omega = \frac{2\pi}{T} = 2\pi f$  where  $T$  is the period

and  $f = \frac{1}{T}$  is the frequency of oscillation. For this particular experiment,  $\omega$  is also known as the

natural frequency and can be determined as  $\omega = \sqrt{\frac{k}{m}}$  where  $k$  is the spring constant and  $m$  is the mass of the body. The constant angle  $\phi$  is called the phase angle because it shifts the sinusoidal curve either right or left.

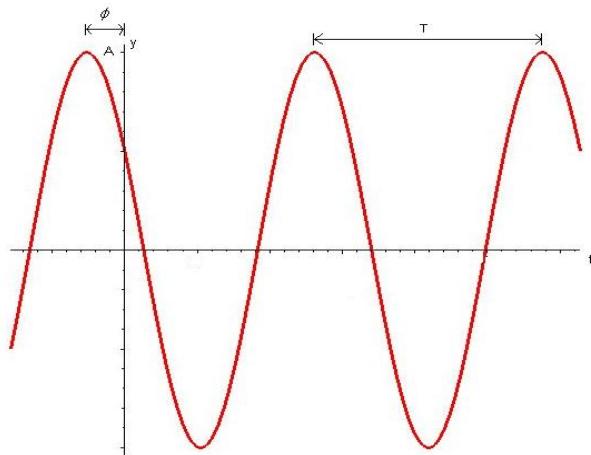


Figure 1 A y-t curve for a body undergoing simple harmonic motion.

A body undergoing harmonic motion will also experience a harmonic (periodic) change in its velocity and acceleration. Thus the velocity ( $v$ ) and acceleration ( $a$ ) will obey the relationships

$$v(t) = -\omega A \sin(\omega t + \phi)$$

$$a(t) = -\omega^2 A \cos(\omega t + \phi)$$

Since the sine and cosine function oscillates between  $\pm 1$ , the extreme values of  $v$  are  $\pm \omega A$  and the extreme values of  $a$  are  $\pm \omega^2 A$ . Therefore the maximum speed and the magnitude of the maximum acceleration of a body moving in simple harmonic motion are

$$v_{\max} = \omega A$$

$$a_{\max} = \omega^2 A$$

In this experiment, you will use an accelerometer to verify characteristics of a mass-spring system moving with harmonic motion.

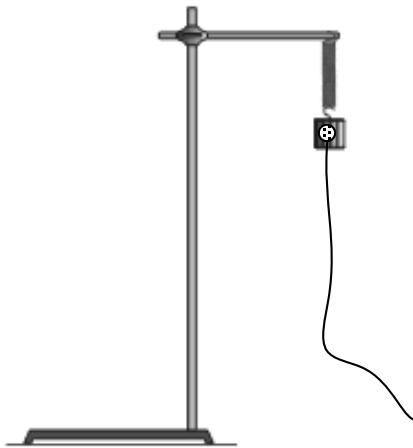


Figure 2

## Objective

- Measure the acceleration as a function of time for an oscillating mass and spring system and compare to a mathematical model of simple harmonic motion.
- Determine the amplitude, period, the spring constant, and the natural frequency of the observed simple harmonic motion.
- Determine whether natural frequency is dependent on mass or initial displacement.

## Equipment List

Windows PC	ring stand, rod, and clamp
LabPro or Universal Lab Interface	spring or rubber band
Logger Pro	tape
Vernier Accelerometer	twist ties
200g and 300g masses	

## Preliminary Questions

1. Attach the 200g mass to the spring (as Procedure Step 1 describes). Pull the mass down about 10cm and release. Observe the motion. Sketch a graph of position *vs.* time for the mass. On the same set of axis, sketch the graphs of velocity *vs.* time and acceleration *vs.* time.

## Procedure

1. Attach the spring to a horizontal rod connected to the ring stand and hang the mass from the spring as shown in Fig 2. Securely fasten the 200g mass to the spring and the spring to the rod using twist ties so the mass cannot fall.
2. Connect the Accelerometer to Channel 1 the LabPro Interface. Open the LoggerPro software. The appropriate graph should appear. Attach the Accelerometer to the mass with a piece of tape
3. Pull the mass down a 3cm ( $A_{initial}$ ) and release. The mass should oscillate along a vertical line only. Click  to begin data collection.
4. After 3s, data collection will stop. The acceleration graph should show a clean sinusoidal curve. The pattern you are observing is characteristic of simple harmonic motion.
5. Compare the acceleration graph to your sketched prediction in the Preliminary Questions. How are the graphs similar? How are they different?
6. Click on the Examine button,  Find the maximum acceleration ( $a_{max}$ ). Next, measure the period ( $T$ ). This can be achieved measuring the time interval between adjacent peak accelerations. Determine the frequency ( $f$ ) and natural frequency ( $\omega$ ). Using  $\omega$ , determine the maximum velocity ( $v_{max}$ ), the maximum displacement ( $A_{computed}$ ), and spring constant ( $k$ ). Record all measurements in the data table.
7. Repeat Steps 3 and 6 with an initial displacement of 6cm ( $A_{initial}$ ) with the same 200g mass. Does the period ( $T$ ), frequency ( $f$ ), or natural frequency ( $\omega$ ) change with larger amplitude?
8. Change the mass to 300g and repeat Steps 3 and 6. Does the period ( $T$ ), frequency ( $f$ ), or natural frequency ( $\omega$ ) change with increasing mass?

## Data Table

Trial	Mass (Kg)	$A_{initial}$ (m)	$a_{max}$ ( $m/s^2$ )	T (s)	f (Hz)	$\omega$ ( $rad/s$ )	$v_{max}$ ( $m/s$ )	$A_{computed}$ (m)	k ( $N/m$ )
1									
2									
3									

## ANALYSIS

1. Where is the mass when the acceleration is zero? Where is the mass when the acceleration is greatest?
2. Does the frequency,  $f$ , appear to depend on the amplitude of the motion? Do you have enough data to draw a firm conclusion?
3. Does the frequency,  $f$ , appear to depend on the mass used? Did it change much in your tests?
4. How does the mass of the accelerometer affect the natural frequency? Does it increase? Does it decrease?