NYU TANDON SCHOOL OF ENGINEERING

NSF

Promoting robotic design and entrepreneurship experiences among students and teachers

Lesson 7: Motion and Dynamics

Innovative Technology Experiences for Students and Teachers (ITEST), Professional Development Program, July 2017-19

Mechatronics, Controls, and Robotics Laboratory, Department of Mechanical and Aerospace Engineering, NYU Tandon School of Engineering

CONTENTS

- Review of basic physics concepts
- Basic physics quantities (scalars and vectors)
- Basic kinematics (displacement, velocity, and acceleration)
- Basic dynamics (force and torque)
- Work, power, energy, mechanical advantage
- Gear and transmission

- **TASK/ACTIVITY:** Gearbox assembly task

BASIC PHYSICS QUANTITIES

- Scalar - quantity with magnitude only
- Vector - quantity with both magnitude (size) and direction

Scalars:

- Distance (100 meters)
- Speed (10 m/sec)
- Time (5 sec)
- Mass (1 kg)
- Energy (100 Joules)

Vectors:

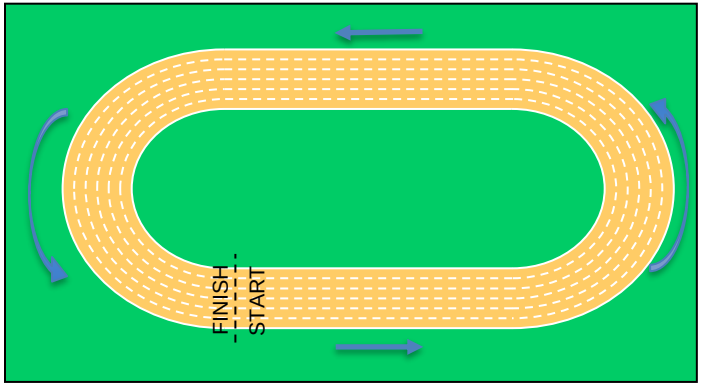
- Displacement (100 meters North)
- Velocity (10 m/sec, NW)
- Acceleration (20 m/sec^2 South)
- Momentum (10 kg•m/s Forward)
- Force (10 N Forward)

DISPLACEMENT

- Displacement (Δx) – the difference between an object's **final position** and its **starting position**

$$\Delta x = x_{final} - x_{initial}$$

- What is the displacement when an athlete makes a complete round on the track shown?



LINEAR VS. ROTATIONAL MOTION

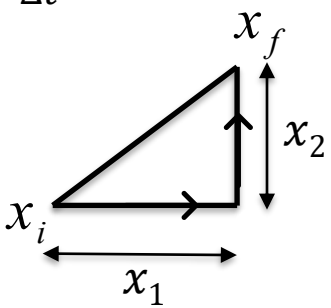
Linear Motion Definitions

- Distance: Δx in meters
- Average speed

$\bar{v} = \frac{\Delta x}{\Delta t}$ in meters/sec

- Average acceleration:

$\bar{a} = \frac{\Delta v}{\Delta t}$ in meters/sec^2



$\Delta x = x_1 + x_2$

Rotational Motion Definitions

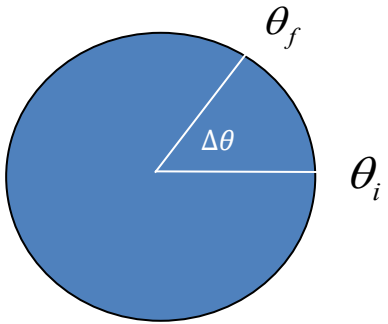
$\Delta\theta$ in radians

- Average angular speed:

$\bar{\omega} = \frac{\Delta\theta}{\Delta t}$ in radians/sec

- Average angular acceleration:

$\bar{\alpha} = \frac{\Delta\omega}{\Delta t}$ in radians/sec^2

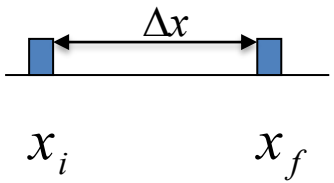


LINEAR VS. ROTATIONAL MOTION

- A car drives 400 m in 20 seconds:

Find the average **speed**

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{400\text{m}}{20\text{s}} = 20\text{m/s}$$



- A wheel spins through an angle of 400π radians in 20 seconds:

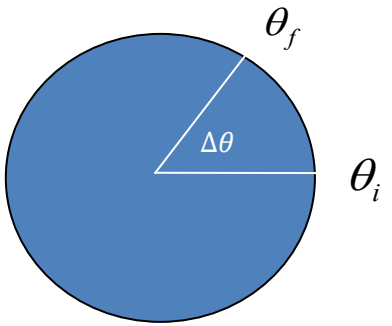
Find the average **angular speed**

$$\bar{\omega} = \frac{\Delta\theta}{\Delta t} = \frac{400\pi \text{ radians}}{20\text{s}}$$

$$\bar{\omega} = 20\pi \text{ rad/sec}$$

$$= 10 \text{ rev/sec}$$

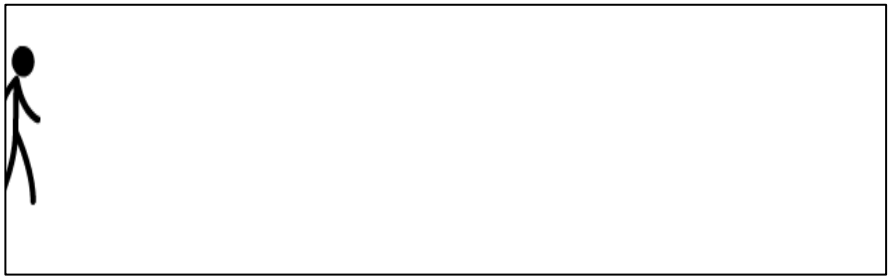
$$= 600 \text{ rev/min}$$



LINEAR VS. ROTATIONAL MOTION

- **Net force** acting on a body produces a **linear acceleration**
  - Linear velocity changes

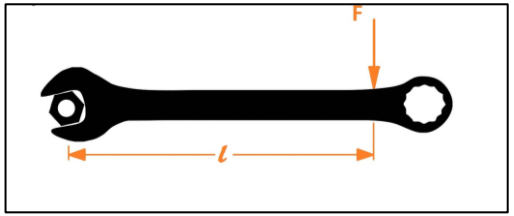
$$a \propto F_{net}$$



Source

- **Net torque** acting on a body produces **angular acceleration**
  - Angular velocity changes (Rate of spin changes)

$$\alpha \propto \tau_{net}$$



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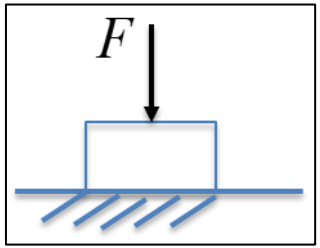
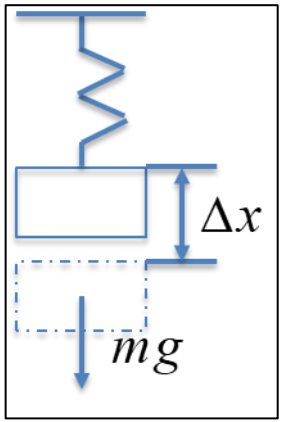
WORK

- Work is equal to the product of applied **force** and the amount of **displacement** along the line of action of that force
- To calculate work done on an object, we compute:

$$\text{Work} = \text{Force} \times \text{Displacement}$$



The work done by F is 0



The work done by gravity is: $mg \times \Delta x$

ENERGY

Energy (E) is defined as the capacity to do **work** *(scalar)*

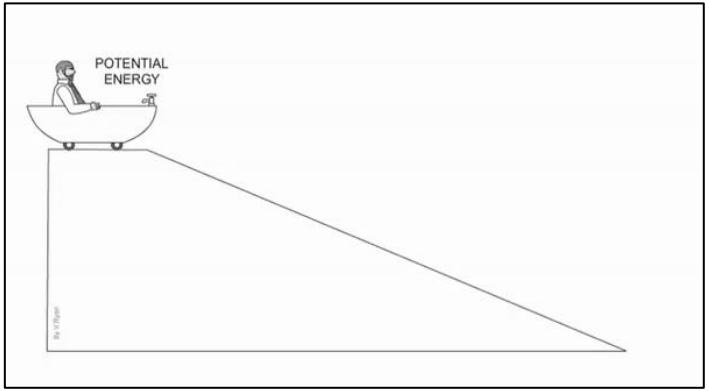
- Many forms of energy exist
  - Not created, only converted
  - Chemical, mechanical etc.

Kinetic Energy (KE):

- Energy due to motion

Potential Energy (PE):

- Energy due to position or deformation
- Unit – Joules (Nm)



Conversion of PE to KE

WORK-ENERGY RELATIONSHIP

- The work done by a net force acting on an object causes a change in the mechanical energy of the object

$$F \times d = \Delta \text{Energy}$$

$$F \times d = \Delta \text{KE} + \Delta \text{PE}$$

POWER

- Power is the rate of doing **work**
- Work = Force x displacement = *F* x *d*

$$\text{Power} = F \times \left(\frac{d}{t} \right) = F \times \bar{v}$$

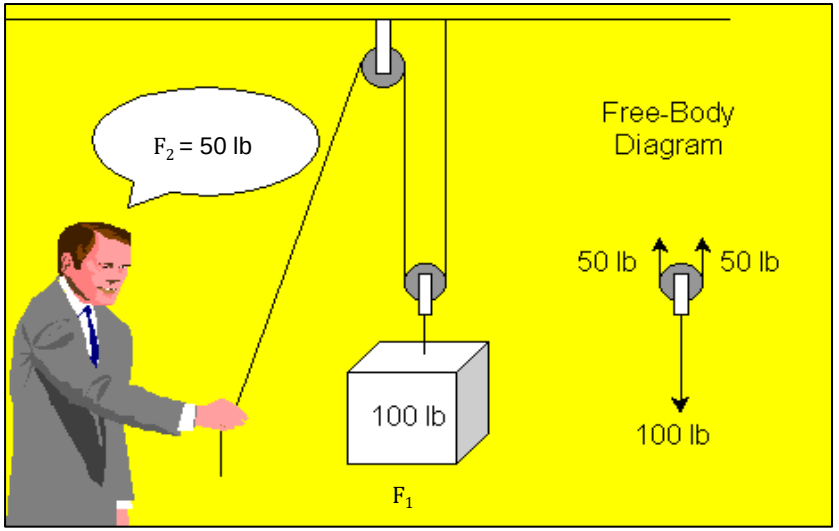
- Units: J/s = watt, 1 Horsepower (HP) = 745.7 watt

WHAT IS MECHANICAL ADVANTAGE (MA)?

- MA is the ratio of the force produced by a machine to the force applied to an object

$$\mathrm{MA} = \frac{\mathrm{F_1}}{\mathrm{F_2}}$$

$\mathrm{F_1}$: The force produced by a machine
$\mathrm{F_2}$: The force applied to the object



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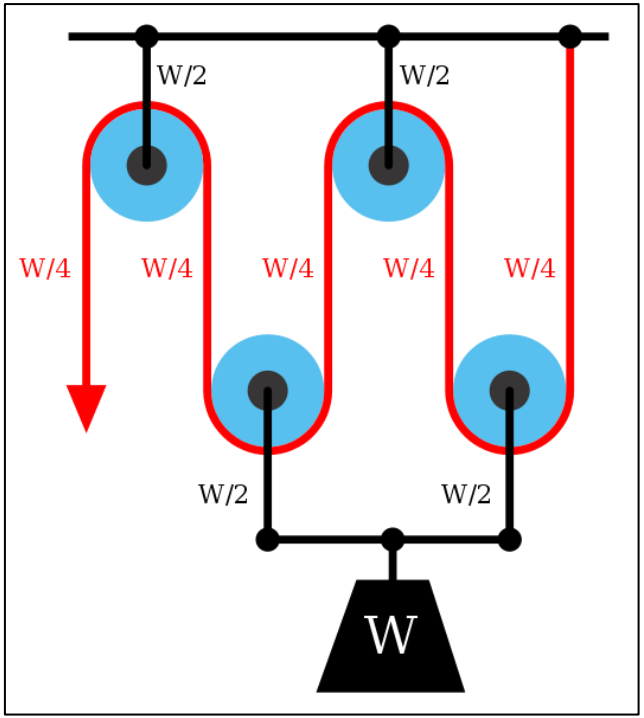
$$\mathrm{MA} = \frac{\mathrm{F_1}}{\mathrm{F_2}} = \frac{100}{50} = 2$$

ACTIVITY - 1

- What is the mechanical advantage if the weight of an object is W?

$$\mathrm{MA} = \frac{\mathrm{F}_1}{\mathrm{F}_2}$$

F_1: The force produced by a machine
F_2: The force applied to the object



Source

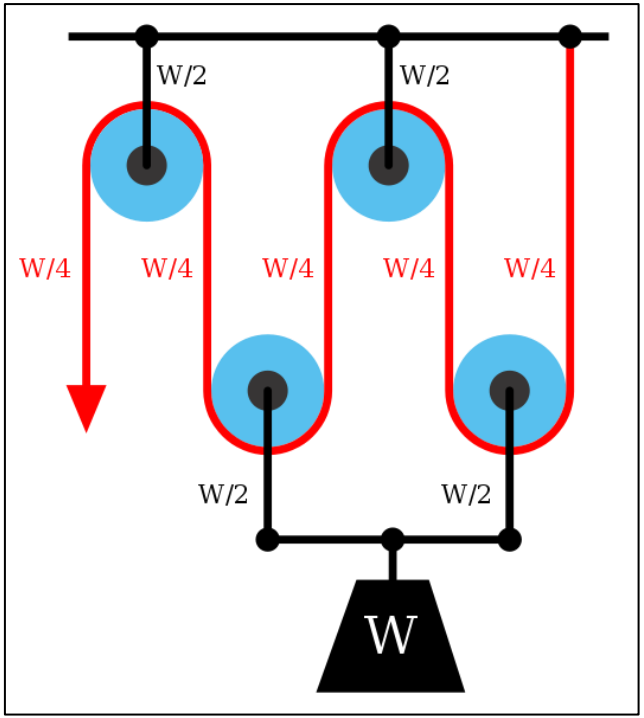
ACTIVITY – 1 SOLUTION

- What is the mechanical advantage if the weight of an object is W?

$$MA = \frac{F_1}{F_2}$$

F_1: The force produced by a machine
F_2: The force applied to the object

$$MA = \frac{F_1}{F_2} = \frac{W}{W/4} = 4$$



Source

GEAR AND TRANSMISSION

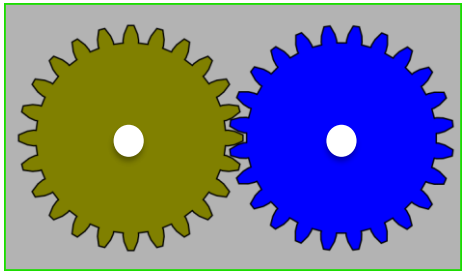
- **Gears –** what are they?
- Gears are the wheels with **teeth**
- Gears **mesh** and **run** the machines
- Gears are used to **transfer motion or power** from one moving part to another

Source

GEARS – PURPOSE

Gears are generally used for one of four different reasons:

- To **reverse** the direction of rotation
- To **increase** or **decrease** the speed of rotation
- To **move** a rotational motion to a different axis
- To keep the rotation of two axes **synchronized**



Spur gears

NEED FOR GEARS

Going downhill:

- Load is **low** with the help of gravity
- Use gear ratio with the **increased speed** at low output torque

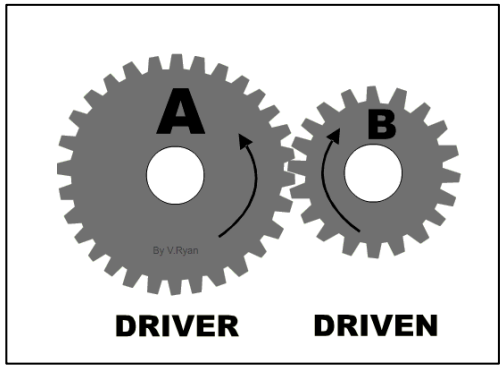
Going uphill:

- Load is **high**
- Use gear ratio with **increased torque** at low speed

Source

SPUR GEARS

- Spur gears are gears in the **same plane** that move **opposite** of each other
  - Gear A: driver gear
  - Gear B: driven gear



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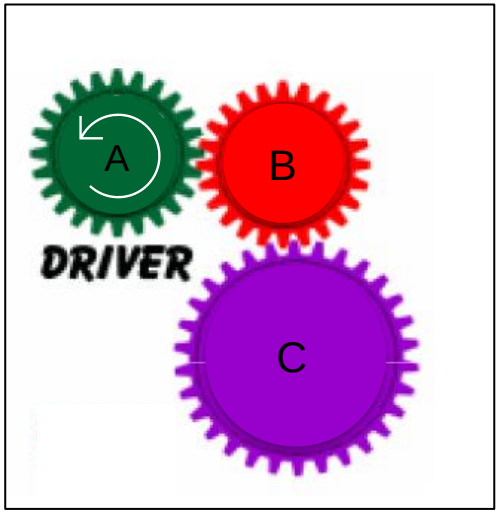
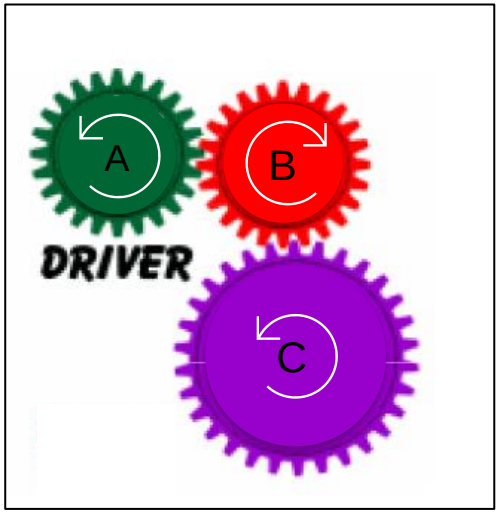
GEAR SYSTEMS



Source

- This is a good example of a gear train
- A gear train is usually made of **two or more** gears
- The driver in this example is gear **A**
- Suppose a motor turns gear A in an anticlockwise direction,
    - Which direction does gear **B** turn?
    - Which direction does gear **C** turn?
    - Does gear **C** revolve faster or slower than gear **A**?

GEAR SYSTEMS



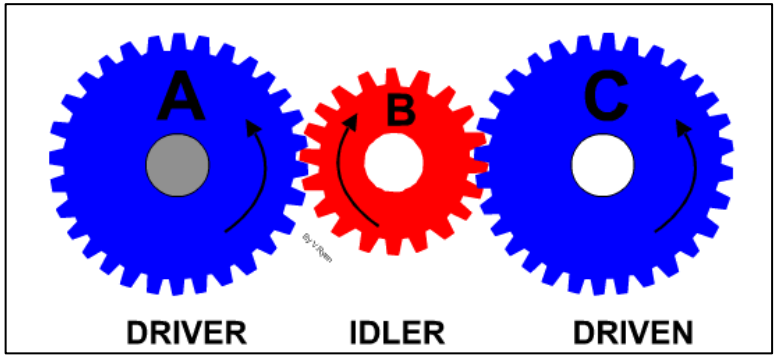
Source

- This is a good example of a gear train
- A gear train is usually made of **two or more** gears
- The driver in this example is gear **A**
- Suppose a motor turns gear A in an anticlockwise direction,
  - Which direction does gear **B** turn? **Clockwise**
  - Which direction does gear **C** turn? **Counter-clockwise**
  - Does gear **C** revolve faster or slower than gear **A**?

  Gear **C** revolves **slower** than Gear **A**

GEAR SYSTEMS

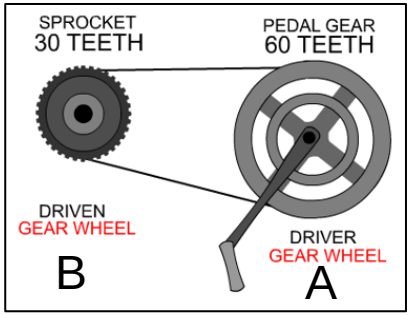
- **Idler gear**: a gear placed between a **driving** and a **driven** gear to transfer motion **without change** of **direction** and **gear ratio**
- Here **gear B** is an idler gear



Source

GEAR RATIO (SPEED RATIO)

- The gear ratio is determined by the **number of teeth** on each gear wheel



Source

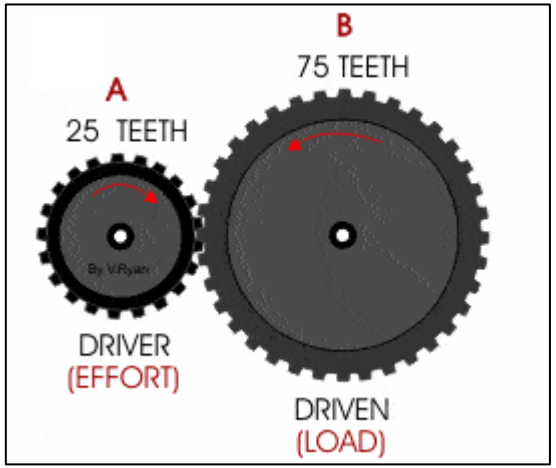
$$\text{Gear Ratio} = \frac{\text{The number of teeth of driven gear}}{\text{The number of teeth of driver gear}} = \frac{n_B}{n_A}$$

$$\frac{\text{The number of teeth of driven gear}}{\text{The number of teeth of driver gear}} = \frac{30}{60} = 1:2$$

- What does this mean?

It means that the driven gear **B** makes **two** rotations for every **one** rotation of the driving gear **A**

GEAR RATIO EXAMPLES



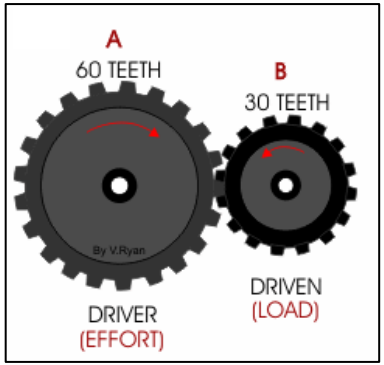
Source

$$\frac{\text{The number of teeth of driven gear}}{\text{The number of teeth of driver gear}} = \frac{75}{25} = 3:1$$

- What does this mean?

It means that the driven gear **B** makes one rotation for every 3 rotations of driving gear **A**

WORKING OUT ROTATIONS PER MINUTE



Source

$$\omega_A n_A = \omega_B n_B$$

Gear A	Gear B
60 teeth	30 teeth
120 rpm	?

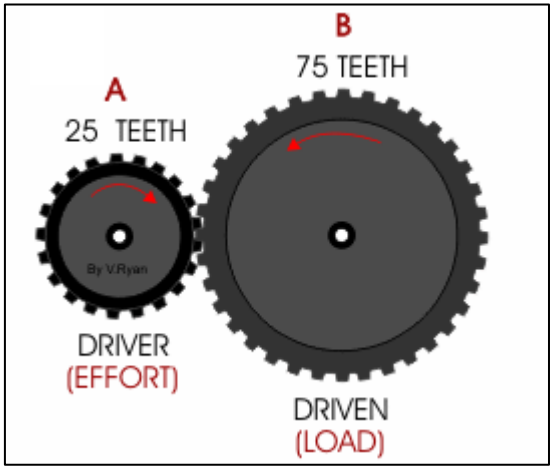
$$\text{Gear Ratio} = \frac{\text{The angular velocity of driver gear}}{\text{The angular veloctiy of driven gear}} = \frac{\omega_A}{\omega_B} = \frac{n_B}{n_A}$$

$$\frac{\omega_B}{\omega_A} = \frac{n_A}{n_B} = \frac{60}{30} = \frac{\omega_B}{120} \qquad \omega_B = 240 \text{ rpm}$$

ACTIVITY - 2

- Calculate the angular velocity of gear **B**

Gear A	Gear B
25 teeth	75 teeth
60 rpm	?

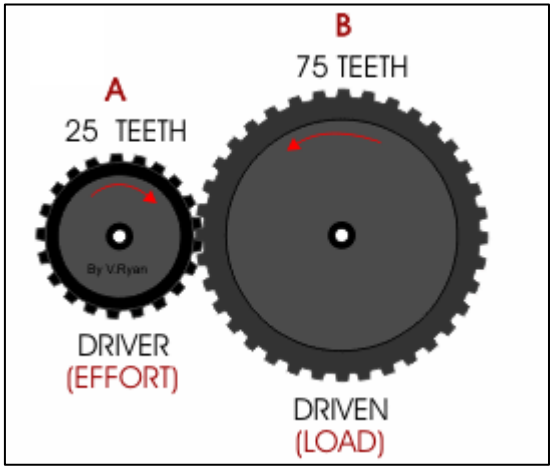


Source

ACTIVITY – 2 SOLUTION

- Calculate the angular velocity of gear **B**

Gear A	Gear B
25 teeth	75 teeth
60 rpm	?



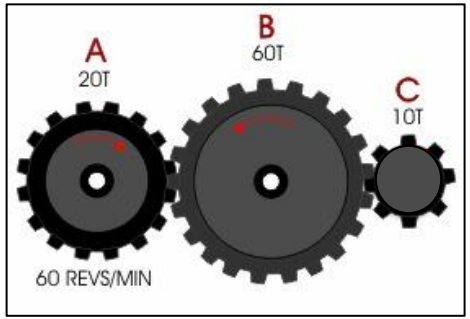
Source

$$\frac{\omega_B}{\omega_A} = \frac{n_A}{n_B} = \frac{25}{75} = \frac{\omega_B}{60}$$

$$\omega_B = 20 \text{ rpm}$$

GEAR RATIOS

- When faced with three gears the question can be broken down into two parts
- First work on gears A and B
- Second work on gears B and C

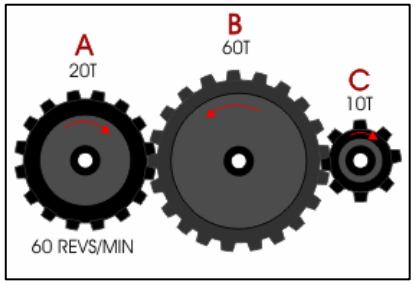


Source

- The diagram shows a gear train composed of three gears
- Gear A revolves at 60 revolutions per minute in a clockwise direction,
  - What is the output in revolutions per minute of gear C?
  - In what direction does gear C revolve?

GEAR RATIOS

START



Source

Gear A	Gear B	Gear C
20 teeth	60 teeth	10 teeth
60 rpm	?	?

$$\frac{\omega_B}{\omega_A} = \frac{n_A}{n_B} = \frac{20}{60} = \frac{\omega_B}{60}$$

$$\omega_B = 20 \text{ rpm}$$

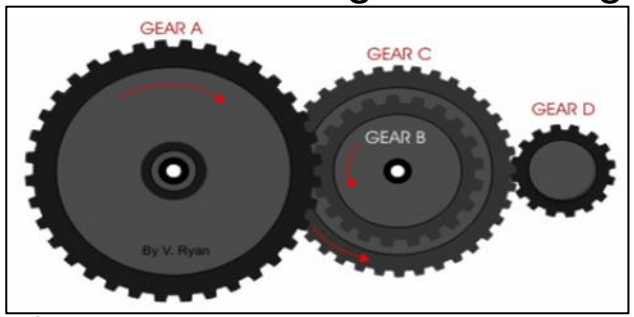
$$\frac{\omega_C}{\omega_B} = \frac{n_B}{n_C} = \frac{60}{10} = \frac{\omega_C}{20}$$

$$\omega_C = 120 \text{ rpm}$$

- Gear C rotates at 120 rpm in a clockwise direction

COMPOUND GEAR RATIOS

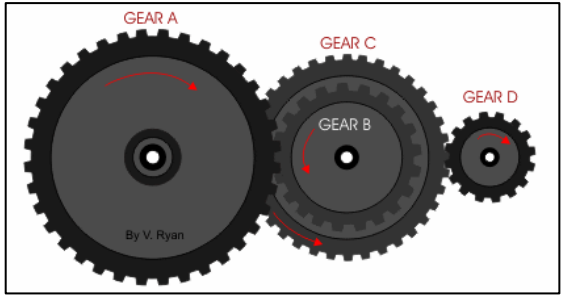
- A compound gear is several gears fixed on the same shaft
- Consequently, they rotate at the **same speed**
- The gears that make up a compound gear usually **differ in size** and have a **different number of teeth**
- Consider the compound gear shown below
  - Split the compound gear into two parts
  - First treat gear A and gear B
  - Next, treat gear C and gear D



Source

- What is the output in revs/min at D?
- What is the direction of rotation of gear D, if gear A rotates in a clockwise direction at 30 revs/min?

COMPOUND GEAR RATIOS



Source

Gear A	Gear B	Gear C	Gear D
120 teeth	40 teeth	80 teeth	20 teeth
30 rpm	?	?	?

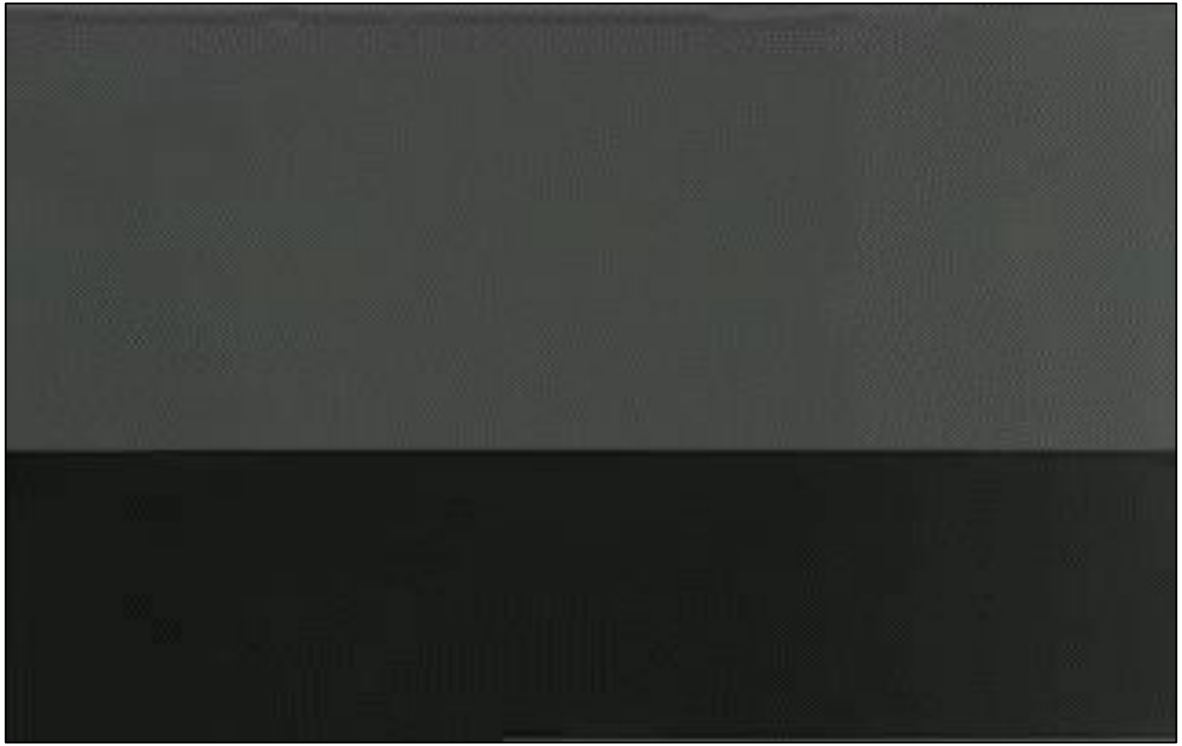
$$\frac{\omega_B}{\omega_A} = \frac{n_A}{n_B} = \frac{120}{40} = \frac{\omega_B}{30}$$

$$\omega_C = \omega_B = 90 \text{ rpm}$$

$$\frac{\omega_D}{\omega_C} = \frac{n_C}{n_D} = \frac{80}{20} = \frac{\omega_D}{90}$$

$$\omega_D = 360 \text{ rpm}$$

HOW DIFFERENTIAL WORKS

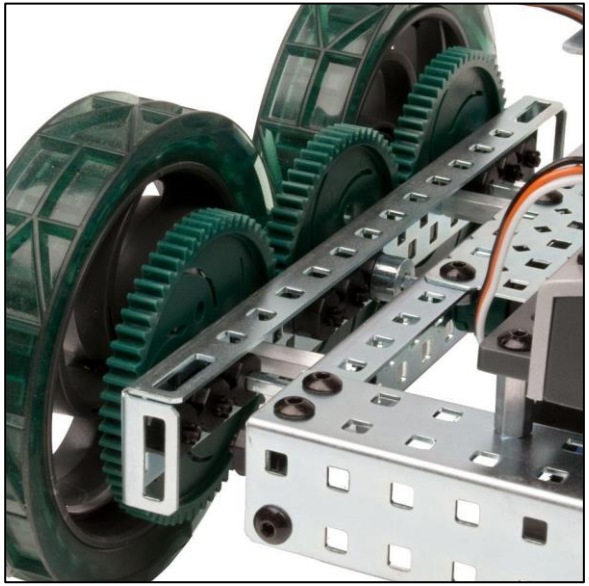


Video

ACTIVITY - 3

- Making a gearbox
- Taking readings and comparing the result with the empirical formula
- Changing internal gearing and finding a new speed

Document



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Thank You!

Questions and Feedback?

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