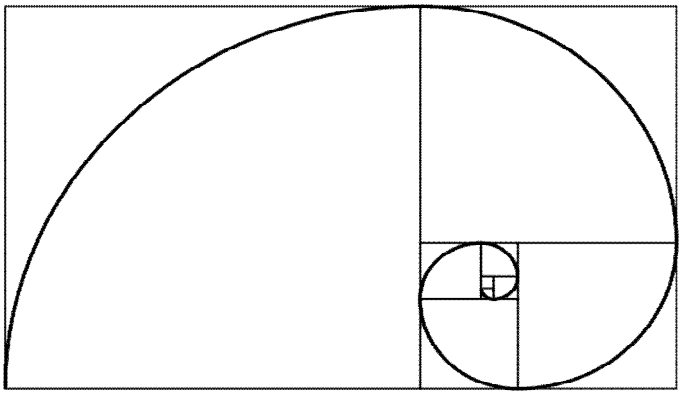
Phi: The Golden Ratio

Subject Areas	Algebra, measurement, numbers, and operations
Associated Unit	None
Associated Lesson	None
Activity Title	Discovering Phi
Header	None
Grade Level	7(6-8)
Activity Dependency	Fibonacci's Robots
Time Required	60 minutes
Group Size	2
Expendable Cost per Group	US$5

Insert Image 1 here, right justified to wrap

Image 1
ADA Description: Fibonacci spiral with squares drawn u to 34
Caption: The Fibonacci Spiral
Image file name: spiral.svg
Source/Rights: Public Domain
http://en.wikipedia.org/wiki/File:Fibonacci_spiral_34.svg



Insert Image 2 here, right justified to wrap

Image 2
ADA Description: Fern frond
Caption: Fern frond unfolding
Image file name: fern.jpg
Source/Rights: Public Domain
http://commons.wikimedia.org/wiki/File:Tree_fern_frond_at_Akatarawa.jpg

Summary

Students discover the mathematical constant phi in two hands-on activities. First, the students measure lengths indicated in pictures of "natural" objects—like bones in the human hand, a nautilus shell, a star—and form ratios of the measured values, which are close to phi. Next, the students learn a basic definition of a mathematical sequence, and specifically the Fibonacci sequence. By taking ratios of successive terms of the sequence, they again find numbers close to phi. This activity is facilitated by using a square puzzle that creates an approximate Fibonacci spiral. Lastly, the instructor implements the rule of the Fibonacci sequence in the Lego robot from the Teach Engineering activity, Fibonacci's Robots, that is equipped with a pen. The robot draws a Fibonacci spiral that is similar to the nautilus.

Engineering Connection

Phi is arguably one of the most important mathematical constants. We use the omnipresence of this number to introduce the students to discrete mathematics. Also, we extend the idea of a mathematical sequence to basic programming using the NXT Mindstorms software.

Engineering Category Relates math concept to engineering

Keywords Fibonacci, Lego, phi, ratio, robotics, sequence

Educational Standards

- New York science: 1.1
- New York math: 3.1, 3.5, 3.7

Pre-Requisite Knowledge Division, basic concept of a function

Learning Objectives

After this activity, students should be able to:

- Describe the general term of the Fibonacci sequence
- Describe examples of Phi in nature
- Identify Phi as the limit of the ratio of terms of the Fibonacci sequence

Materials List

Each group needs:

- Ruler
- Calculator
- Laminated Fibonacci square puzzle made from poster board

To share with the entire class:

- Lego NXT kit, with robot constructed and programmed as in Teach Engineering activity Fibonacci's Robots
- Several copies of laminated natural pictures
- Marker
- Tape
- Large sheets of paper

Introduction/Motivation

The mathematical constant phi has been studied since at least the 300 B.C.E., when it was defined by the Greek mathematician Euclid. The formalization of phi may have been motivated by its presence in the pentagram, a common religious symbol at that time. Its ubiquity in nature, from snails' shells and flowers' seed heads, explains its key role into Western aesthetics. Phi, as a ratio features prominently in the works of da Vinci and Dali. Moreover, phi has been observed in many areas of mathematics, from geometry to dynamical systems. The purpose of this activity is to allow the students to "discover" phi in two ways: through simulated empirical discovery and through examination of a specific mathematical object. Thus, they find phi both experimentally and theoretically.

The mathematical source of phi, the Fibonacci sequence, is an intuitive sequence formed by adding two successive terms to get the next term. This makes it a natural first sequence for students to encounter. In the square puzzle part of this activity, students can also spontaneously solve this puzzle in a variety of ways by making increasingly larger rectangles with side lengths

terms of the Fibonacci sequence. This part of the activity naturally leads to the discovery that the ratio of successive terms of the Fibonacci sequence is phi.

The integration of robotics into this activity synthesizes what the students find in their examination of the Fibonacci sequence with the rigidity of computer programming. The students are able to recognize that the robot does only exactly what is it told to do, incapable of making addition errors for example. This encourages an engineer's precision on the part of the students, and also gives credence to the Fibonacci spiral as a vehicle of phi as the robot draws the same spiral when programmed with the general term of the sequence.

Vocabulary / Definitions

Word	Definition
Sequence (numerical)	An ordered set of numbers arranged according to a rule.
Term	One of the numbers in a sequence
Limit of a sequence	The number approached by the terms of a sequence as one proceeds through the order of the sequence

Procedure

Before the Activity

- Construct and program the Lego NXT robot as in the Teach Engineering activity, Fibonacci's Robots.
- Tape a marker to robot so that the marker will draw on the floor as the robot moves.
- Secure large sheets of paper to floor.
- Print out and laminate natural pictures attached to this document. There should be enough copies so that each group can have at least one picture at any one time.
- Cut poster board as in attached diagram, labeling size on each piece, and laminate. There should be one puzzle for each group.
- Copy attached worksheet so that there is one for every two students.

With the Students

1. Divide the class into pairs.
    a. Distribute to each group, a ruler, a calculator, a worksheet, a square puzzle, and one natural picture.
2. Introduce the concept of a mathematical constant, specifically phi. You may use the aforementioned history anecdotally. Also, remember that the students may already be familiar with the constant pi, and this can be a good starting point for a discussion on constants.
3. Display each of the natural pictures to the entire class and name each: the hand, the nautilus, and the star. Each of these natural objects can be considered as harboring the golden ratio, phi.
    a. Have each team designate a measurer and a recorder.
    b. Considering their nature picture, the measurer must measure the indicated lengths A, B, C, and D and relay these numbers to the recorder.
    c. The recorder writes down the values on the worksheet, and fills in the rest of Part I of the worksheet with the ratios of the lengths using the calculator.

    d. The teams trade pictures and repeat this activity until all groups have considered all four pictures.
4. Discuss the students' results. What was the trend of the ratios found in the pictures?
5. Now introduce the concept of a mathematical sequence, and specifically the Fibonacci sequence, using the aforementioned history.
    a. Go through the first four or five terms of the sequence with the class, and construct the first two or three pieces of the square puzzle.
    b. Let the pairs complete Part II of the worksheet together, finding more terms of the Fibonacci sequence and also ratios of successive terms.
    c. Let the students complete the square puzzle.
    d. Discuss their worksheet answers to relate the presence of phi in this mathematical object.
    e. Discuss the relationship of the lengths of the sides of the squares in the square puzzle and the Fibonacci sequence.
6. Bring the class back together for a final discussion. If possible, display the amended Mindstorms code from Fibonacci's Robots, which has the robot proceed distances dictated by the Fibonacci sequence and turn at right angles. If the students are unfamiliar with the software, explain the components of the program.
    a. Set up the robot on sheets of paper fixed to the floor, with the marker attached so that a line is drawn as the robot moves.
    b. Run the program as the class observes.
    c. Discuss how, using the robot, we can visually perceive phi imbedded in the Fibonacci sequence.

Attachments

Natural Pictures (pdf)

Natural Pictures (ppt)

Square Puzzle Template (pdf)

Square Puzzle Template (ppt)

Worksheet (pdf)

Worksheet (doc)

Robot Program (rbt)

Troubleshooting Tips

Test the robot's program before the class, and make sure the paper is large enough to have the robot draw at least a couple turns of the spiral.

Investigating Questions

None

Assessment

Pre-Activity Assessment
Guessing game: Tell students there is a number that they "like", even if they know which number it is. Describe natural and artistic situations where phi can be found. Ask for suggestions of what they think the number may be, and have each pair record their guess on their copy of the worksheet.

Activity Embedded Assessment
Fibonacci spiral: After explaining the robot's program, ask the students to predict the shape that the robot will draw.

Post-Activity Assessment
Tuning the sequence: Challenge the students to think about would happen if we wanted the robot to travel a different initial distance. Would it change the ratio that previously gave us phi?

Activity Extensions
None

Activity Scaling
- For upper grades, have students construct and program the robot, as in Activity Fibonacci's Robots, using the concept of variables developed in that activity. Also, discuss how phi appears in the ratios of the terms of the Fibonacci sequence independent of the initial conditions. Illustrate this using the robot, by changing starting distances.

Additional Multimedia Support
The Golden Ratio by Mario Livio

References
Livio, Mario. The Golden Ratio. New York: Broadway Books, 2002.

Redirect URL http://gk12.poly.edu/amps/

Owner Nicole Abaid

Contributors Nicole Abaid, Alexander Kozak

 The development of this activity was supported by Project AMPS under a GK-12 Fellows grant 0741714 from the National Science Foundation.

Version: August, 2009